Size-dependent and Anisotropic Coulomb failure criterion

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ABSTRACT: This paper proposes the size-dependent and anisotropic Coulomb failure criterion to describe the size effect and anisotropy of shale strength. The failure criterion assumes that cracks and bedding planes cause size effect and anisotropy. The failure criterion has six parameters, the cohesion, friction angle, scaling exponent related to crack and bedding plane. The cohesion decreases as rock size increases based on the scaling exponents while the friction angle is independent of rock size. This paper validates the failure criterion from three aspects. Strength data of anisotropic rock is collected from published research. The failure criterion "fits" the collected strength data of anisotropic rock. A bonded-particle model of shale is developed in PFC2D which explicitly includes cracks and bedding planes. The model strength fits well with the failure criterion. Finally, experiments conducted in shale rock validates the hypothesis that the variation of cohesion and friction angle with rock size follows the failure criterion.

Keywords: Size effect, Anisotropy, Coulomb failure criterion, Griffith theory, Single plane of weakness model.

1 INTRODUCTION

The underground coal mines in the Appalachian region have suffered from the highest risk of roof falls in the US (Bajpayee et al. 2014). The roof falls are mainly due to the presence of shale in roof (Murphy 2016). To prevent roof fall, it is imperative to know shale strength when designing an excavation layout and roof support. The strength of shale is affected by two important factors: size and anisotropy. Herein size refers to the diameter of a cylinder or the width of a rectangular prism. Figure 1 (a) presents the typical variation of rock strength relative to rock size (D). The strength decreases as D increases from laboratory scale to in-situ scale and the decrease rate diminishes. Figure 1 (b) shows the typical variation of rock strength relative to the bedding plane angle β. The strength is maximum when β is close to 0 and 90°, and the strength is minimum when β is around 60°. This strength variation with β is referred to as the "U-shaped" curve. However, the knowledge about the combination of anisotropy and size effect on shale strength is not clear.

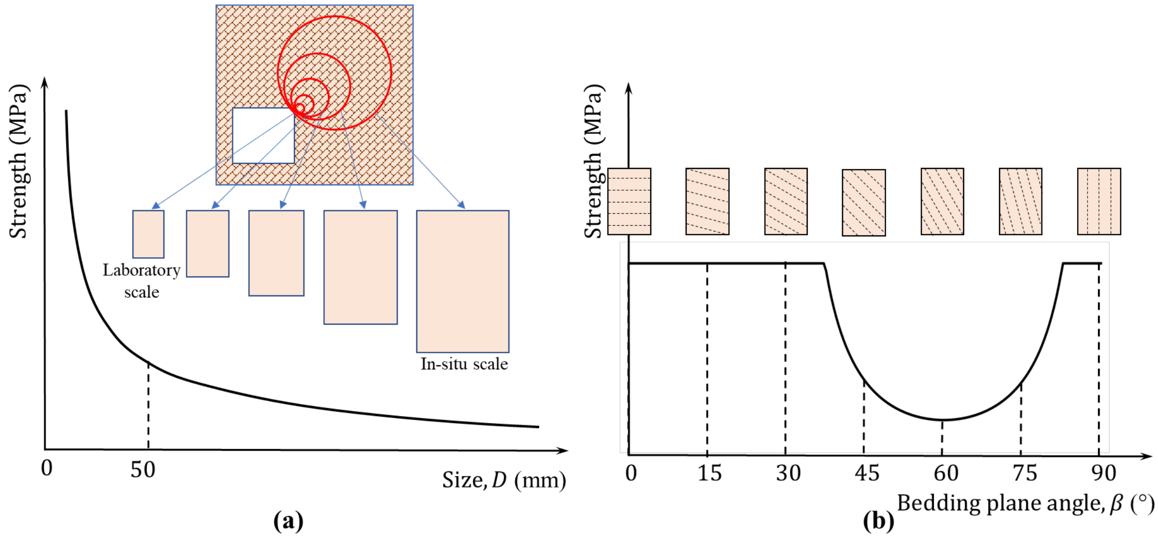

Figure 1. Variation of rock strength relative to the rock size D and bedding plane angle β.

Shale is made up of several thin laminas that are formed in the sedimentary process (O'Brien 1996). The bedding planes exist between laminas. In addition, there are small-scale cracks inside shale matrix (Ambrose 2014). Therefore, the structure of shale can be simplified as consisting of shale matrix, bedding planes, and cracks, as shown in Figure 2. The Griffith criterion and Coulomb criterion can predict failure caused by cracks and bedding planes respectively. McClintock & Walsh (1962) modified the Griffith criterion to relate the strength of rock to the length of cracks under compression, in which case cracks are closed and there is friction effect along the crack surface. Jaeger (1960) modified the Coulomb criterion to describe the anisotropic strength of rock that contains a set of parallel bedding planes. Based on the work of McClintock, Walsh and Jaeger, this paper proposes the size-dependent and anisotropic Coulomb failure criterion. This failure criterion can describe the strength of shale and other anisotropic rocks with similar structure.

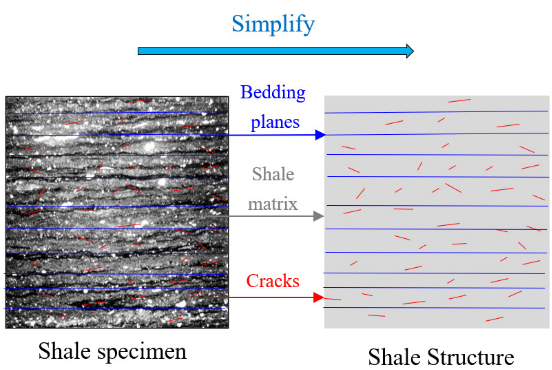

Figure 2. Simplified shale structure consisting of shale matrix, bedding planes, and cracks.

2 FAILURE CRITERON FORMULATION

This paper assumes that shale matrix is isotropic and size-independent, bedding planes cause anisotropy, and cracks cause size effect. Based on this assumption, the Coulomb failure criterion is used to describe the strength behavior of shale matrix. When bedding planes are introduced into shale matrix, failure can occur inside the shale matrix or along bedding planes. The strength becomes anisotropic. The single plane of weakness model (Jaeger 1960) can describe the strength behavior. When randomly distributed cracks are introduced into shale matrix, failure will initiate at one of the cracks rather than shale matrix. According to McClintock & Walsh (1962), the strength is determined by the crack with largest length, which is also related to model size. Zhao et al. (2023) derived the size-dependent Coulomb failure criterion to relate the strength of shale matrix with cracks, see Eq. (1). Due to the page limit, the derivation and verification are excluded.

$$\sigma_1 = \frac{2c_{c50}\left(\frac{D}{50}\right)^{-k_c}\cos\phi_c + \sigma_3(1+\sin\phi_c)}{1-\sin\phi_c} \tag{1}$$

where the cohesion c_{c50} , the friction angle ϕ_c, and the scaling exponent k_c are related to cracks. The cohesion decreases as size increases while the friction angle is constant.

When both bedding planes and cracks are considered, as shown in Figure 3 (a), the failure can take place at bedding planes or cracks. Following the single plane of weakness model, the size-dependent and anisotropic Coulomb failure criterion was proposed as Eq. (2).

$$\sigma_1 = \min\begin{cases}\dfrac{2c_{w50}\left(\frac{D}{50}\right)^{-k_w} + \sigma_3(u_w - u_w\cos 2\beta_w + \sin 2\beta_w)}{-u_w - u_w\cos 2\beta_w + \sin 2\beta_w} \\ \dfrac{2c_{c50}\left(\frac{D}{50}\right)^{-k_c}\cos\phi_c + \sigma_3(1+\sin\phi_c)}{1-\sin\phi_c}\end{cases} \tag{2}$$

where the bedding plane angle β_w, the cohesion c_{w50} , the friction coefficient u_w, and the scaling exponent k_w are for bedding planes. Figure 3 (b) shows how the strength varies with D, β_w, and σ_3. One can determine the parameters of the proposed failure criterion based on the triaxial compression test of shale specimens with different size and orientation.

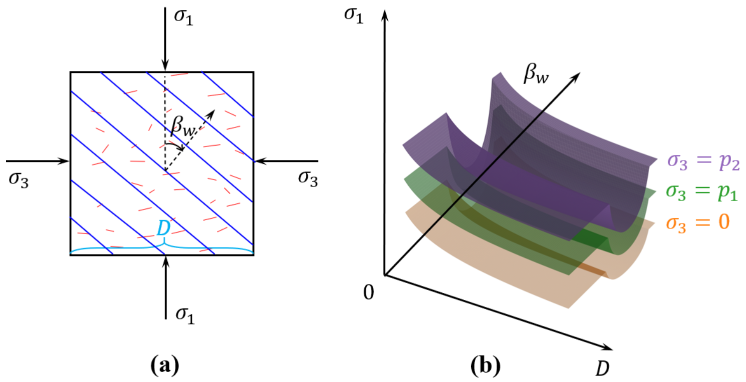

Figure 3. Shale in the two-dimensional stress condition and its strength surface.

It is important to note that the proposed failure criterion is of general type and can be modified to other three cases: if anisotropy (bedding planes) does not exist, the criterion reduces to Eq. (1), which is size-dependent and isotropic failure criterion; if size effect (cracks) does not exist, the criterion reduces the single plane of weakness model, which is size-independent and anisotropic; If neither anisotropy (bedding planes) or size effect (cracks) exists, the criterion reduces to the Coulomb failure criterion, which is size-independent and isotropic.

3 VERIFICATION

3.1 Experimental data

This section aims to collect experimental data about the size effect and anisotropy of rock strength to validate the proposed failure criterion. Only one dataset about the compressive strength of slate is found from Li et al. (2021). Figure 4 shows the fitting result using Eq. (2). Each point represents the compressive strength of slate with specific size D, bedding plane angle β, and confining stress σ_3. The colored surface represents the fitting surface. The parameters c_{c50}, ϕ_c, and k_c are 26.96 MPa, 44.10°, and 0.22 respectively. The parameters c_{w50}, ϕ_w (ϕ_w is the friction angle of bedding plane, $\phi_w = \arctan u_w$), and k_w are 23.39 MPa, 27.79°, and 0.47 respectively. The coefficient of determination R^2 is 0.866. The failure criterion fits well with the strength of slate.

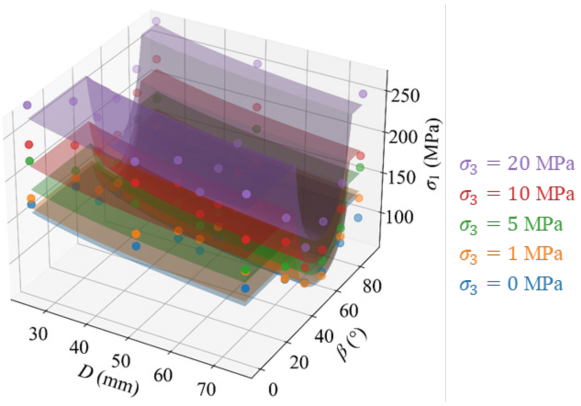

Figure 4. Size effect and anisotropy of the compressive strength of slate and the fitting result.

3.2 *Numerical modeling*

This section aims to establish a numerical model which incorporates size effect and anisotropy, use the numerical model to test the assumptions of the failure criterion, and use the model strength to validate the failure criterion. PFC2D is used to model shale matrix, bedding planes, and cracks by bonded particles, smooth joints, and discrete fractures respectively. The assembly of bonded particles, smooth joints, and discrete fractures forms the bonded-particle model of shale (BPM-Shale), as shown in Figure 5.

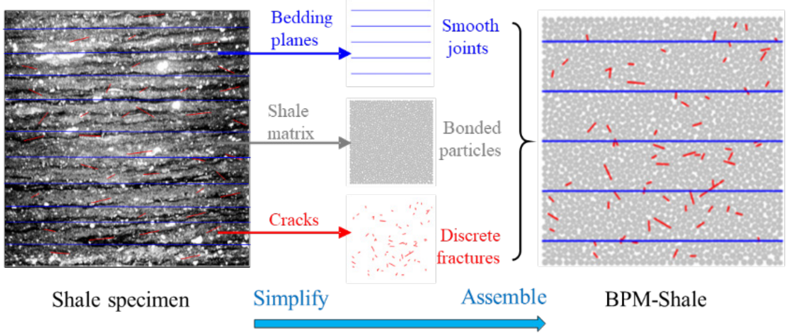

Figure 5. Structure of shale and PFC2D modelling technique

Figure 6 shows the main steps of the model development. Step 1 generates the model of shale matrix (BPM-Matrix). The strength of BPM-Matrix is size-independent and isotropic. Step 2 creates the model of shale matrix with bedding planes (BPM-Bedding) by inserting smooth joints into the BPM-Matrix. The strength of BPM-Bedding becomes size-independent and anisotropic. Step 3 creates the model of shale matrix with cracks (BPM-Crack) by inserting the discrete fractures into the BPM-Matrix. The strength of BPM-Crack becomes size-dependent and isotropic. Step 4 creates BPM-Shale by adding the smooth joints and the discrete fractures into the BPM-Matrix. The strength of BPM-Shale becomes size-dependent and anisotropic. Due to the page limit, the model generation and calibration in detail is not presented here. The proposed model and its development procedure are applicable to other anisotropic rocks for studying the size effect and anisotropy (Zhao et al., 2022).

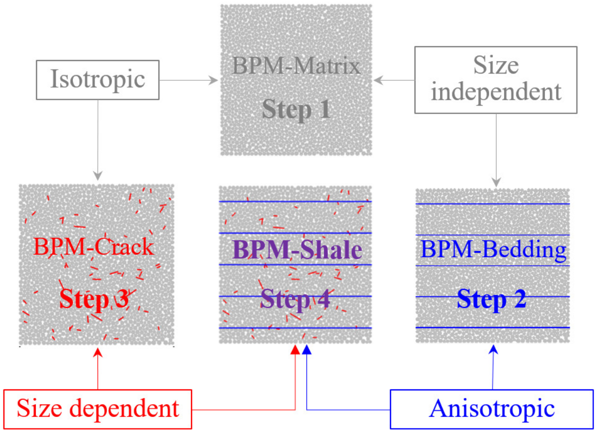

Figure 6. Development procedure of the Bonded-particle model of shale.

This study creates the BPM-Shale at different D (25, 75, 100, 125, 150, and 200 mm) and at different β (0, 15, 30, 45, 60, 75, and 90°). These models are tested in the unconfined and confined compression test with σ_3 being 3, 5, and 10 MPa. Figure 7 presents the compressive strength of BPM-Shale with specific size D, bedding plane angle β, and confining stress σ_3. The strength exhibits the "U-shaped" curve against β at different D and different σ_3. In addition, the strength shows the decreasing trend against D at different β and different σ_3, and the decrease rate diminishes. Therefore, the compressive strength of BPM-Shale shows anisotropy and size effect. The colored surface represents the fitting surface. The cohesion c_{c50}, the friction angle ϕ_c, and the scaling exponent k_c are 10.58 MPa, 26.34°, and 0.21 respectively; the cohesion c_{w50}, the friction angle ϕ_w, and the scaling exponent k_w are 7.80 MPa, 15.96°, and 0.26 respectively; the coefficient of determination is $R^2 = 0.810$. Therefore, the failure criterion fits well with the strength of BPM-Shale.

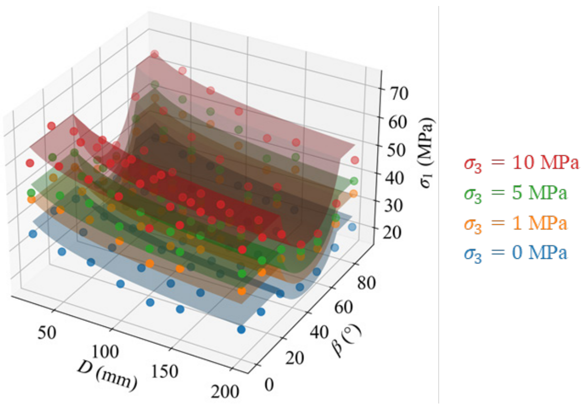

Figure 7. Size effect and anisotropy of the compressive strength of BPM-Shale and the fitting result.

3.3 *Experimental evidence*

There is experimental evidence that supports the proposed failure criterion. On one hand, Tani (2001) investigated the size effect of the strength of mudstone and found that the cohesion decreased as the specimen size increased following $c_{cD} = c_{c50}(D/50)^{-k_c}$, with $k_c = 0.5$. Kong et al. (2021) investigated the size effect of the strength of sandstone and determined $k_c = 0.221$. In addition, their works showed that the friction angle was independent of rock size. On the other hand, Barton (1990) showed that the shear strength of joints (or bedding planes) decreased as joint length increased. Similar results have been reported by Pratt (1974), Cunha (1990), and Ohnishi & Yoshinaka (1995). In other words, the cohesion of bedding plane might decrease as the specimen size increased. These works can more or less validate the proposed failure criterion.

4 CONCLUSION

This paper proposes the size-dependent and anisotropic Coulomb failure criterion to describe the strength behavior of shale considering size effect and anisotropy. The failure criterion contains six parameters. The cohesion, friction angle, and scaling exponent of cracks; the cohesion, friction angle, and scaling exponent of bedding planes. Once the parameters are determined through data fitting, the criterion can predict the strength of shale at different size, orientation, and confining stress. Moreover, the proposed failure criterion is of general type and can be modified to different situations in which anisotropy or size effect are not existent.

The proposed failure criterion is verified by using experimental data, numerical modeling, and experimental evidence. The failure criterion demonstrates that bedding planes cause anisotropy and cracks cause size effect. The cohesion of bedding planes and cracks decreases as rock size increases while the friction angle is constant.

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REFERENCES

Ambrose, J. (2014). *Failure of Anisotropic Shales under Triaxial Stress Conditions*. Ph.D. thesis. Imperial College London.

Bajpayee, T. S., Pappas, D. M., & Ellenberger, J. L. (2014). Roof Instability: What Reportable Noninjury Roof Falls in Underground Coal Mines Can Tell Us. *Professional Safety*, 59(3), 57–62.

Barton, N. (1990). Scale effects or sampling bias. *Scale Effects in Rock Mechanics*, 31–55.

Cunha, A. P. (1990). *Scale Effects in Rock Masses: Proceedings of the First International Workshop on Scale Effects in Rock Masses, Loen, Norway, 7-8 June 1990.* AA Balkema.

Jaeger, J. C. (1960). Shear failure of anisotropic rocks. *Geological Magazine*, 97(1), 65–72.

Kong, X., Liu, Q., & Lu, H. (2021). Effects of Rock Specimen Size on Mechanical Properties in Laboratory Testing. *Journal of Geotechnical and Geoenvironmental Engineering*, 147(5), 04021013.

Li, K., Yin, Z.-Y., Han, D., Fan, X., Cao, R., & Lin, H. (2021). Size Effect and Anisotropy in a Transversely Isotropic Rock Under Compressive Conditions. *Rock Mechanics and Rock Engineering*, 54(9), 4639–4662.

McClintock, F. A., & Walsh, J. (1962). Friction on Griffith's cracks in rock under pressure. *US National Congress on Applied Mechanics.*

Murphy, M. M. (2016). Shale Failure Mechanics and Intervention Measures in Underground Coal Mines: Results From 50 Years of Ground Control Safety Research. *Rock Mechanics and Rock Engineering*, 49(2), 661–671.

O'Brien, N. R. (1996). Shale lamination and sedimentary processes. *Geological Society, London, Special Publications*, 116(1), 23–36.

Ohnishi, Y., & Yoshinaka, R. (1995). Laboratory investigation of scale effect in mechanical behavior of rock joint. *Fractured and Jointed Rock Masses*, 465–470.

Pratt, H. R. (1974). Friction and deformation of jointed quartz diorite. *Proc. 3rd ISRM Congress, Denver*, 1974, 306–310.

Tani, K. (2001). Scale effect on shear strength of sedimentary soft rocks observed in triaxial compression test (influence of potential joints). *36th National Conference of the Japanese Geotechnical Society*, 597–598.

Zhao, Y., Mishra, B., Shi, Q., & Zhao, G. (2022). A Size-Dependent Bonded-Particle Model for Transversely Isotropic Rock and its Application in Studying the Size Effect of Shale. Geotechnical and Geological Engineering, 40(11), 5437–5453.

Zhao, Y., Mishra, B., Shi, Q., & Zhao, G. (2023). Size-dependent Mohr–Coulomb failure criterion. Bulletin of Engineering Geology and the Environment, 82(6), 218.